Fundamental phase noise on supercontinuum generated in microstructure fiber: limits on frequency metrology

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Abstract: We present quantitative results for the excess phase noise of a frequency comb that employs microstructure fiber. This fundamental phase noise arises from amplification of the input shot noise during supercontinuum formation in the fiber.
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Broad phase-stabilized frequency combs can be generated by launching femtosecond pulses from a Ti:Sapphire laser into nonlinear microstructure fiber and phase-locking the resulting output frequency comb [1, 2]. While these combs have exhibited remarkable stability [1-6], noise on the comb will ultimately limit the metrological applications. There are two major noise seeds on the input laser pulse that give rise to noise across the frequency comb: shot noise and low-frequency "technical" noise. The amplitude noise on the supercontinuum arising from these noise seeds has been quantified [7-9], and the effect of these noise seeds on compression and coherence has been considered [10-12]. In Ref. [13], the carrier-envelope offset (CEO) phase noise induced by the laser technical noise was measured. However, there have been no general predictions of the phase noise (in particular on the repetition frequency) induced during supercontinuum formation. Here we derive a general expression that describes both amplitude and phase noise on the comb generated by any input noise seed. We specifically quantify the phase noise floor resulting from the amplification of input shot noise, since this noise represents a fundamental limit. As with the amplitude noise, we find that significant phase noise is generated from the input pulse shot noise during supercontinuum formation in the nonlinear fiber.
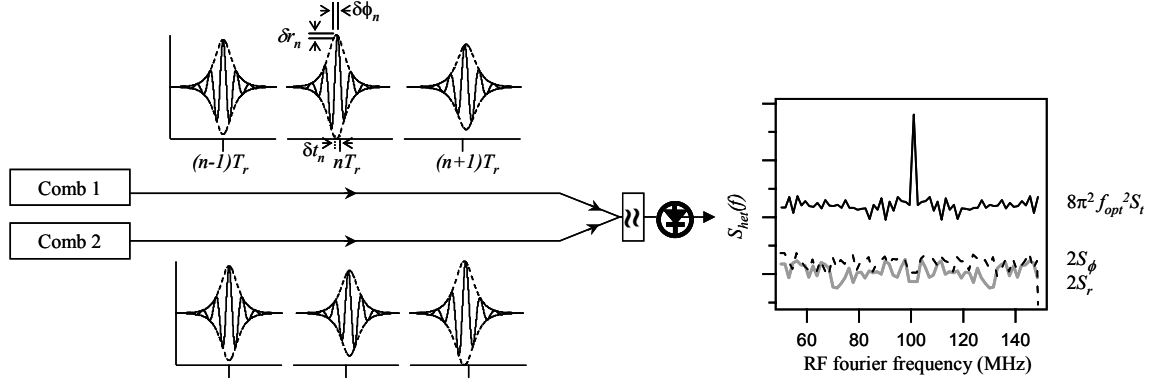

Fig 1. Basic setup analyzed. The three time-domain perturbations from amplified shot noise each contribute to the white-noise floor of the RF heterodyne spectrum as detailed in Eq. (2) below.

We consider the situation described in Fig. 1. The supercontinuum output is numerically simulated using the generalized nonlinear Schrödinger equation for multiple input pulses with different noise realizations to mimic the input pulse train [7, 8]. The electric field of the nth spectrally-filtered pulse versus time, t, can be written in general as

$$E_n(t) = (1 + \delta r_n(t)) e^{i\delta\phi_n(t)} e^{in\Delta\phi_{CEO}} E_0(t - nT_r - \delta t_n),\qquad(1)$$

where $\Delta\phi_{CEO}$ is the CEO phase shift per pulse and T_r is the inverse of the repetition rate f_r. The three small perturbations δr_n, $\delta\phi_n$ and δt_n are respectively the amplitude noise, phase noise on the comb offset frequency, and timing jitter (or phase noise on f_r). If the corresponding power spectral densities (PSDs) of the pulse-averaged quantities are, S_r, S_ϕ and S_t, the output RF spectrum for the heterodyne beat from two distinct combs (with identical mean values) at a central optical frequency f_{opt} is

$$S_{het}(f) \sim \sum_n \delta(f - nf_r) + 2\pi^2 f^2 S_t(f - nf_r) + 2S_r(f - nf_r) + 2S_\phi(f - nf_r) + 8\pi^2 f_{opt}^2 S_t(f - nf_r). (2)$$

Figure 2 shows one example of the three noise PSDs across the supercontinuum that results from shot noise on the input pulse. Based on these data, the dominant contribution arises from the last timing jitter term; for the conditions of Fig. 2, the timing jitter is ~ 0.75 fs and gives rise to a white-noise floor of ~-72 dBc/Hz. For a homodyne measurement (comb 1=comb 2 in Fig. 1), Eq. (2) is modified so that only the first two noise terms contribute (and are multiplied by two). In this case, the amplitude noise dominates; for the conditions of Fig. 2, the amplitude noise is ~0.7% and gives rise to a white-noise floor of ~-120 dBc/Hz, as in Refs. [7, 9]. Finally, the amplitude noise and timing jitter are intimately connected since they both arise from the underlying electric-field fluctuations.
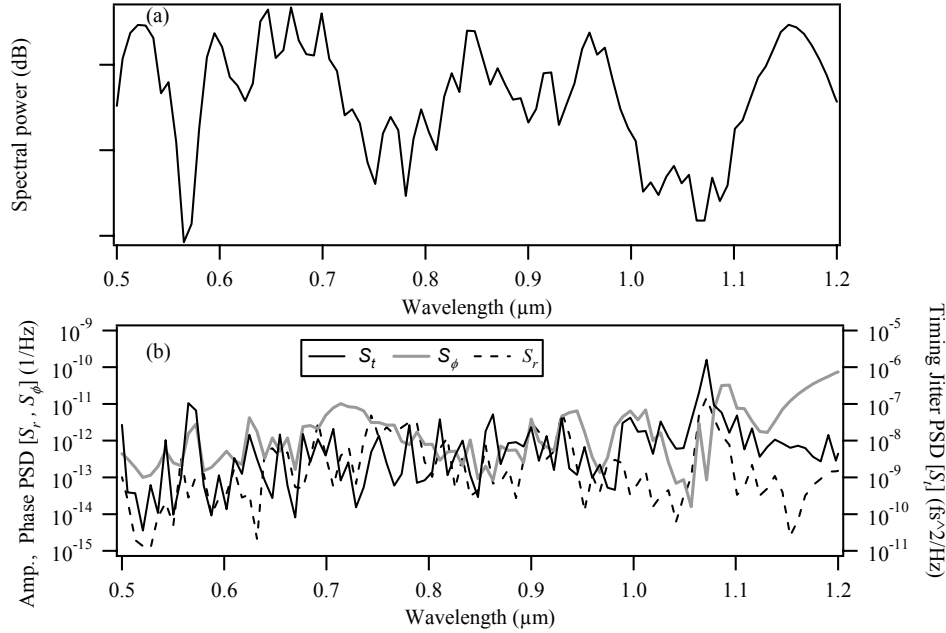

Fig. 2. (a) Simulated supercontinuum generated in 15 cm of microstructure fiber by a 0.9 nJ pulse with a spectral width of 49 nm and chirp of 260 fs^2. (b) The broadband amplitude noise (S_r), CEO phase noise (S_ϕ), and timing jitter noise (S_t) assuming an 8 nm filter bandwidth.

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